1988 Q2. H.L.

(a) Two boats move with constant speed 5 m/s relative to the water and both cross a straight river of width 72 m flowing with constant speed 3 m/s parallel to the banks. One crosses by the shortest path and the other in the shortest time. Show that the difference in the times taken is 3·6 s.

(b) Two ships A and B move with constant speeds $2u$ and u respectively. At a certain instant, B is 2400 m due east of A and moving northwards. Show that A must move in the direction 30° North of East in order to intercept B and find (in terms of u) the time it takes to intercept B.

(a) <u>shortest path</u> $(V_{BW} > V_{WG})$
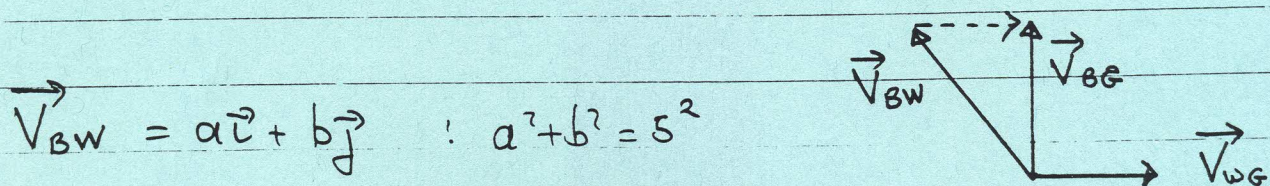
$$\vec{V_{BW}} = a\vec{i} + b\vec{j} \quad : \quad a^2 + b^2 = 5^2$$

$$\vec{V_{BG}} = \vec{V_{BW}} + \vec{V_{WG}} \qquad : \quad V_{WG} = 3\vec{i}$$

$$= (a+3)\vec{i} + b\vec{j}$$

For shortest Route $\quad \vec{V_{BG}} \cdot \vec{i} = 0$

$$\Rightarrow \quad \underline{a = -3} \quad : \quad \underline{b = 4}$$

$$\underline{\text{Time to cross}} = \frac{72}{4} = 18 \text{ secs}.$$

<u>shortest time</u> (heads straight across)

$$\underline{\text{shortest Time}} = \frac{72}{5} = 14\frac{2}{5} \text{ sec}$$

$$\text{Difference in times} = 18 - 14\frac{2}{3}$$

$$= 3\frac{3}{5} \text{ sec}$$